Simultaneous Contact, Gait and Motion Planning for Robust Multi-Legged Locomotion via Mixed-Integer Convex Optimization

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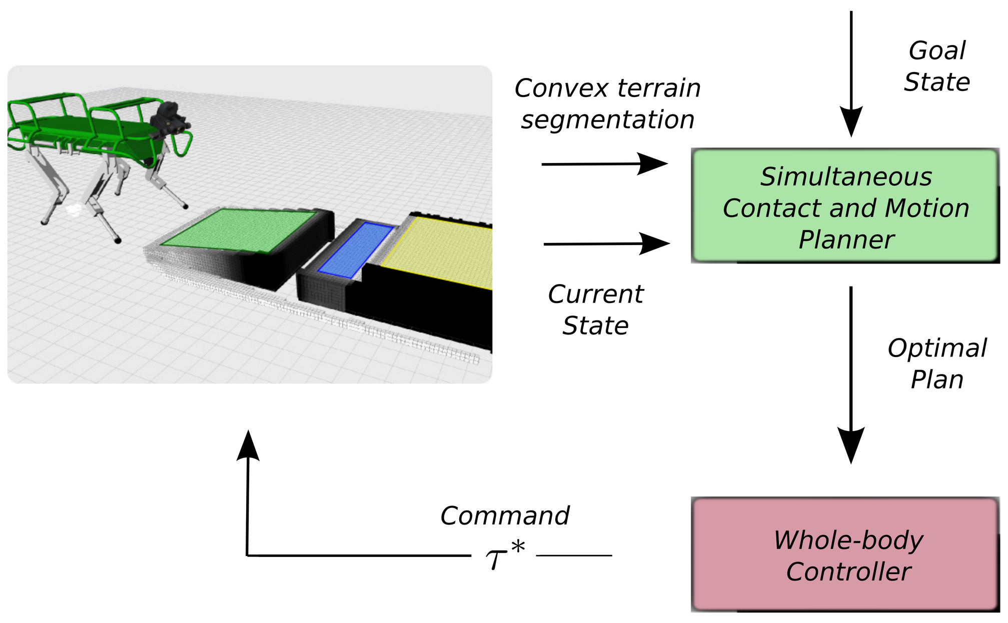
Introduction

Reasoning about contacts and motions simultaneously is crucial for generating complex whole-body behaviors. We propose a mixed-integer convex formulation to plan simultaneously contact locations, gait transitions and motion, in a computationally efficient fashion.

Approach Overview



Trajectory Optimization

We formulate a convex trajectory optimization:

$$\min_{\mathbf{r},\mathbf{k}_o,\mathbf{p}_l,\boldsymbol{\lambda}_l} g_T + \sum_{k=1}^{N} g(k) \tag{1}$$

The running cost $g(k)$ maximizes the stability of the motion, while seeking for the fastest and smoothest gait:

$$g(k) = \|\ddot{\mathbf{r}}_k\|_{\mathbf{Q}_v} + \|\boldsymbol{\lambda}_{l,k}\|_{\mathbf{Q}_F} + q_u \mathbf{U}_k + q_t t_k - q_\alpha \alpha_k, \tag{2}$$

1. Minimize the CoM acceleration $\ddot{\mathbf{r}}$.
2. Minimize the contact forces magnitude $\|\boldsymbol{\lambda}\|$.
3. Minimize the upper bound of quadratic terms $\mathbf{U} = (\mathbf{u}^-, \mathbf{u}^+)$.
4. Maximize the stability margin α.
5. Minimize the execution time.

The terminal cost g_T biases the plan towards its goal ($\mathbf{r}_G$):

$$g_T = \|\mathbf{r}_T - \mathbf{r}_G\|_{\mathbf{Q}_g} \tag{3}$$

In practice, we add a small cost to $\|\dot{k}\|_{Q_k}$ in order to generate smoother motions.

Whole-body Control

Reference CoM acceleration ($\ddot{\mathbf{r}}^r \in \mathbb{R}^3$) and body angular acceleration ($\dot{\boldsymbol{\omega}}_b^r \in \mathbb{R}^3$) through a *virtual model*:

$$\begin{aligned}\ddot{\mathbf{r}}^r &= \ddot{\mathbf{r}}^d + \mathbf{K}_r(\mathbf{r}^d - \mathbf{r}) + \mathbf{D}_r(\dot{\mathbf{r}}^d - \dot{\mathbf{r}}),\\ \dot{\boldsymbol{\omega}}_b^r &= \dot{\boldsymbol{\omega}}_b^d + \mathbf{K}_\theta e(\mathbf{R}_b^d \mathbf{R}_b^T) + \mathbf{D}_\theta(\omega_b^d - \omega_b),\end{aligned} \tag{4}$$

where $\mathbf{K}_r, \mathbf{D}_r, \mathbf{K}_\theta, \mathbf{D}_\theta \in \mathbb{R}^{3\times 3}$ are positive-definite diagonal matrices of proportional and derivative gains, respectively.

We formulate the tracking problem using QP with the generalized accelerations and contact forces as decision variables, $\mathbf{x} = [\ddot{\mathbf{q}}^T, \boldsymbol{\lambda}^T]^T \in \mathbb{R}^{6+n+3n_l}$:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} g_{err}(\mathbf{x}) + \|\mathbf{x}\|_{\mathbf{W}}; \quad \mathbf{Ax} = \mathbf{b}, \quad \underline{\mathbf{d}} < \mathbf{Cx} < \bar{\mathbf{d}} \tag{5}$$

$$g_{err}(\mathbf{x}) = \left\| \begin{matrix} \ddot{\mathbf{r}} - \ddot{\mathbf{r}}^r \\ \dot{\boldsymbol{\omega}}_b - \dot{\boldsymbol{\omega}}_b^r \end{matrix} \right\|_{\mathbf{S}} \tag{6}$$

The equality constraints $\mathbf{Ax} = \mathbf{b}$ encodes dynamic consistency, stance condition and swing task. The inequality constraints $\underline{\mathbf{d}} < \mathbf{Cx} < \bar{\mathbf{d}}$ encode friction, torque, and kinematic limits [1].

We map the optimal solution $\mathbf{x}^*$ into desired feed-forward joint torques $\boldsymbol{\tau}_{ff}^* \in \mathbb{R}^n$:

$$\boldsymbol{\tau}_{ff}^* = \begin{bmatrix}\mathbf{M}_{bj}^T & \mathbf{M}_j\end{bmatrix} \ddot{\mathbf{q}}^* + \mathbf{h}_j - \mathbf{J}_{cj}^T \boldsymbol{\lambda}^* \tag{7}$$

These are summed with the joint PD torques (i.e. feedback torques $\boldsymbol{\tau}_{fb}$) to form the desired torque command $\boldsymbol{\tau}^d$:

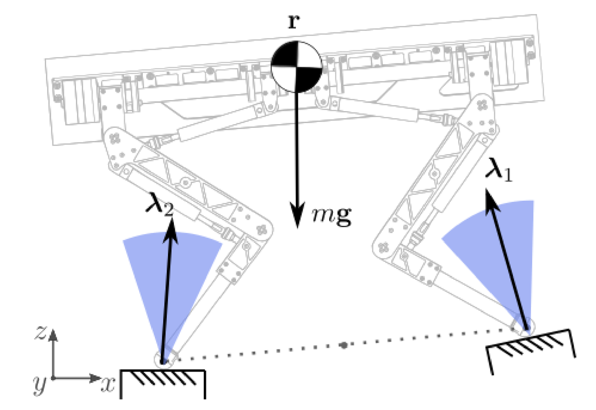
$$\boldsymbol{\tau}^d = \boldsymbol{\tau}_{ff}^* + PD(\mathbf{q}_j^d, \dot{\mathbf{q}}_j^d), \tag{8}$$

which is sent to a low-level joint-torque controller.

Simultaneous Contact and Motion Planning

A. Centroidal Dynamics

$$\begin{bmatrix} m\ddot{\mathbf{r}} \\ \dot{\mathbf{k}} \end{bmatrix} = \begin{bmatrix} m\mathbf{g} + \sum_{l=1}^{n_l} \boldsymbol{\lambda}_l \\ \sum_{l=1}^{n_l} (\mathbf{p}_l - \mathbf{r}) \times \boldsymbol{\lambda}_l \end{bmatrix}, \tag{9}$$

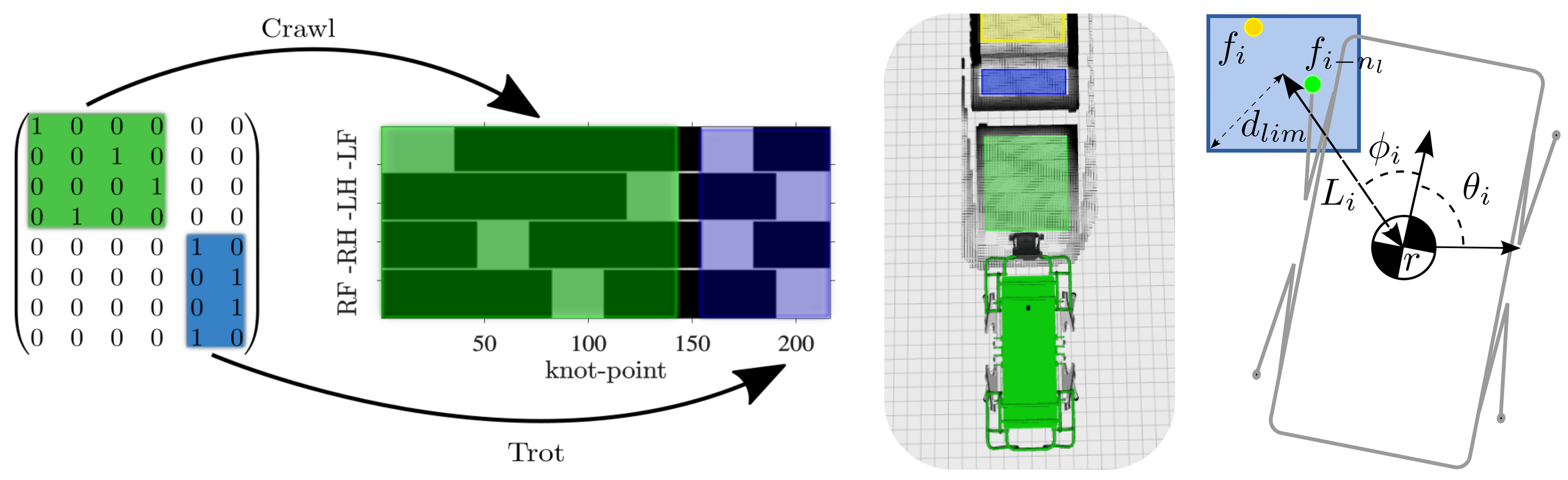


- CoM position $\mathbf{r}$
- Angular Momentum $\mathbf{k}$
- Contact force of end-effector $\boldsymbol{\lambda}_l$
- Position of end-effector $\mathbf{p}_l$

where $\mathbf{p}_l - \mathbf{r}$ can be described as bilinear function [2] and decomposed as:

$$\mathbf{ab} = \frac{\mathbf{u}^+ - \mathbf{u}^-}{4} \quad \mathbf{u}^+ \geq (\mathbf{a}+\mathbf{b})^2 \quad \mathbf{u}^- \geq (\mathbf{a}-\mathbf{b})^2. \tag{10}$$

B. Gait Sequence



A gait matrix [3] $\mathbf{T} \in \{0,1\}^{N_f \times N_t}$ where $\mathbf{T}_{ij} = 1$ means that the robot will move to the i^{th} contact location at the j^{th} time-slot.

- number of contacts N_f
- number of time slots N_t

Each contact location is reached once: $\sum_{j=1}^{N_t} \mathbf{T}_{ij} = 1$, $\forall i = 1, .., N_f$.

C. Contact Location

We constrain the contacts to lie within one of N_r *convex safe contact surfaces* (each represented as a polygon $\mathcal{R} = \{\mathbf{c} \in \mathbb{R}^3 | A_r \mathbf{c} \leq b_r\}$).

After assigning the contact to swing, we optimize the contact locations $\mathbf{f} = (f_x, f_y, f_z, \theta)$ and we assign this contact to one of the N_r using a binary matrix:

$$\sum_{r=1}^{N_r} \mathbf{H}_{ir} = 1, \quad \mathbf{H}_{ir} \Rightarrow A_r \mathbf{f}_i \leq b_r \tag{11}$$

We approximate the kinematic limits as a bounding box with respect to the CoM:

$$\left\| \mathbf{f}_i - \left[\mathbf{r}_{T(i)} + L_i \begin{pmatrix} \cos(\theta_i + \phi_i) \\ \sin(\theta_i + \phi_i) \end{pmatrix} \right] \right\| \leq d_{lim}, \tag{12}$$

- CoM position at i transition $\mathbf{r}_{T(i)}$
- Diagonal of the bounding box $d_{\lim}$
- Distance from the trunk of leg L_i

(the trigonometric functions are decomposed in piecewise linear functions) [4].

D. End-effector Trajectories

We define $\gamma(j,t)$ as swing reference trajectory, connected to adjacent contacts, where:

- j indicates the time-slot
- $t \in [1, \dots, N_k]$ all the knots per time-slot

The leg reaches the contact position $\mathbf{f}_i$ at the end of the j slot:

$$\mathbf{T}_{ij} \Rightarrow \mathbf{p}_{l(i)\gamma(j,N_k)} = \mathbf{f}_i, \tag{13}$$

where $l(i)$ is the leg number for the i^{th} contact.

To constrain that the leg remains stationary when there is no transition:

$$\sum_{i \in C(l)} \mathbf{T}_{ij} = 0 \Rightarrow \mathbf{p}_{l\gamma(j,t)} = \mathbf{p}_{l\gamma(j,1)} \; \forall t \in [2, \dots, N_k], \tag{14}$$

where $C(l)$ are the contact indexes assigned to the l^{th}leg.

We ensure kinematic feasibility by constraining the CoM position with respect to the end-effectors (bounding box constraint):

$$\mathbf{d}_- < \mathbf{r}_j - \frac{\sum_{l=1}^{n_l} \mathbf{p}_{lj}}{n_l} < \mathbf{d}_+ \tag{15}$$

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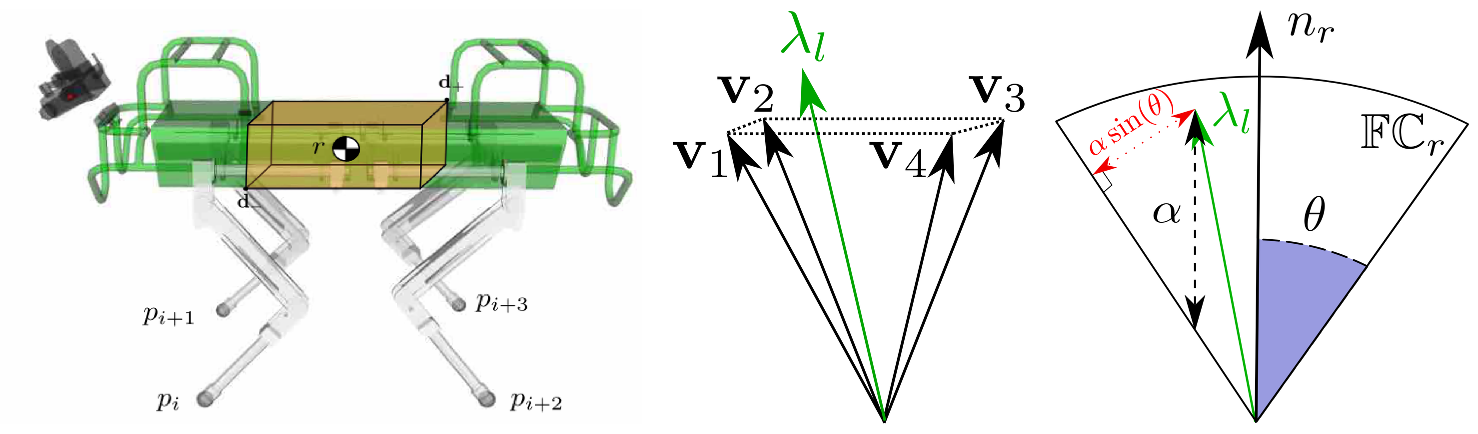
Simultaneous Contact and Motion Planning



E. Contact Dynamics

If the l^{th} leg is in swing mode, there is no contact force:

$$\sum_{i\in C(l)} \mathbf{T}_{ij} = 1 \Rightarrow \boldsymbol{\lambda}_{l\gamma(j,t)} = 0\ ,\ \forall t \in NC(j), \tag{16}$$

where $NC(j)$ is the set of knots in the j^{th} slot used for the swing (complementarity constraint [5]).

Stability in non-coplanar contact conditions:

$$\boldsymbol{\lambda}_{lj} \in \mathbb{FC}_{\mathrm{r}} \Rightarrow \boldsymbol{\lambda}_{lj} = \sum_{e=1}^{N_e} \rho_e \mathbf{v}_{\mathrm{r}_e}\ ,\ \ \rho_1, \dots, \rho_{N_e} > 0,$$

where ρ_e are positive multipliers on each cone edge.

To add robustness to the motion, we maximize the distance between the nonlinear friction cone boundary and the force vector:

$$\alpha = \arg\max_{\bar{\alpha}}\ \text{s.t}\ \boldsymbol{\lambda}_{lj} - \bar{\alpha}\hat{\mathbf{n}}_{\mathrm{r}} \in \mathbb{FC}_{\mathrm{r}},$$

We introduce the following linear constraint over each safe surface:

$$\mathbf{T}_{ij} \text{ and } \mathbf{H}_{ri} \Rightarrow \boldsymbol{\lambda}_{l(i)\gamma(j)} - \alpha_{l(i)\gamma(j)}\hat{\mathbf{n}}_{\mathrm{r}} \in \mathbb{FC}_{\mathrm{r}}\ ,\ \ \alpha \geq 0. \tag{17}$$

Since the contact cone must not change when it is in stance phase, we also add the constraint:

$$\sum_{l\in C(i)} \sum_{t\in NS(j)} \mathbf{T}_{lt} = 0,$$

$$\Rightarrow \boldsymbol{\lambda}_{l(i)\gamma(j)} - \alpha_{l(i)\gamma(j)}\mathbf{n}_{\mathrm{r}} \in \mathbb{FC}_{\mathrm{r}}, \tag{18}$$

where $NS(j)$ is the set of time-slots succeeding j.

G. Approximate Torque Limits

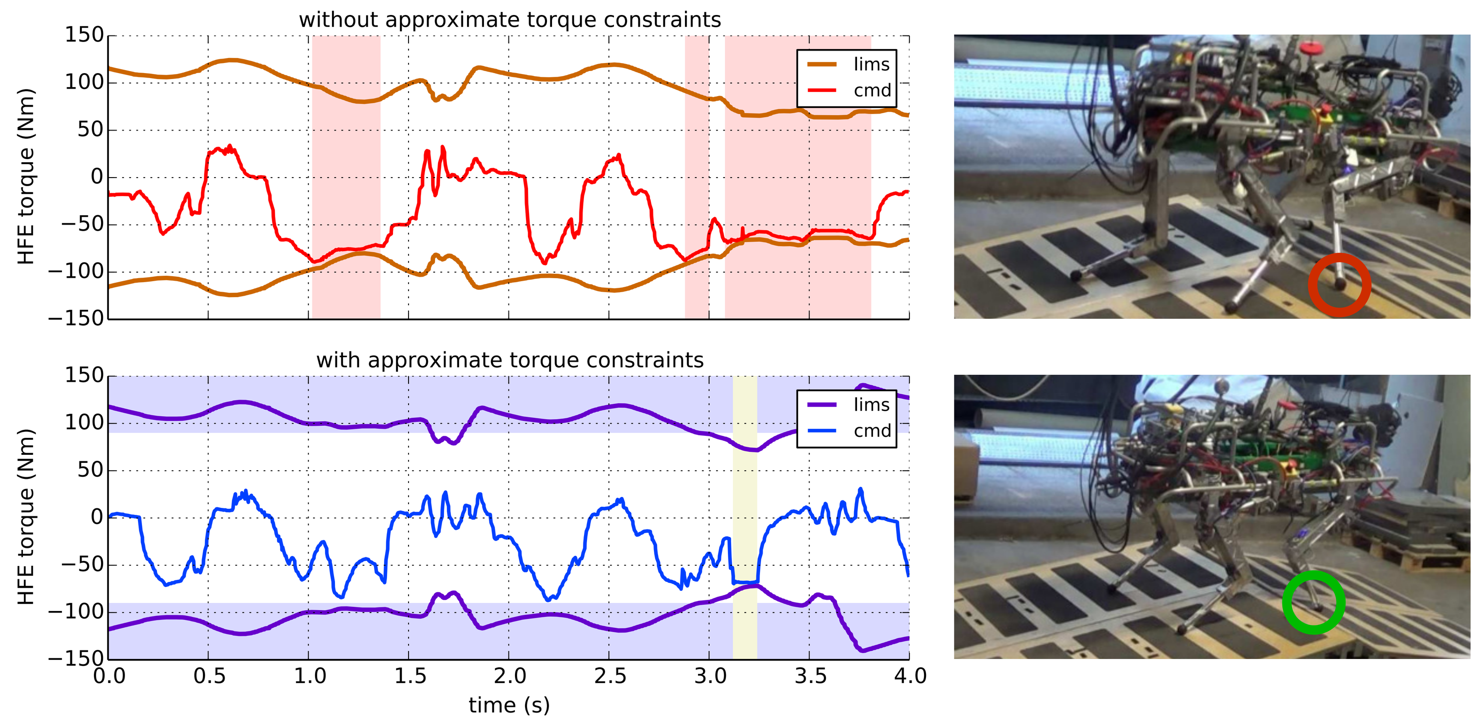
We approximate the torque limits using a quasi-static motion assumption:

$$\mathbf{J}_{l,j}^T \boldsymbol{\lambda}_{l,j} \leq \boldsymbol{\tau}_{max},\quad \Rightarrow \quad \mathbf{J}^{*T}_{l,j}\boldsymbol{\lambda}_{l,j} \leq \boldsymbol{\tau}_{max},$$

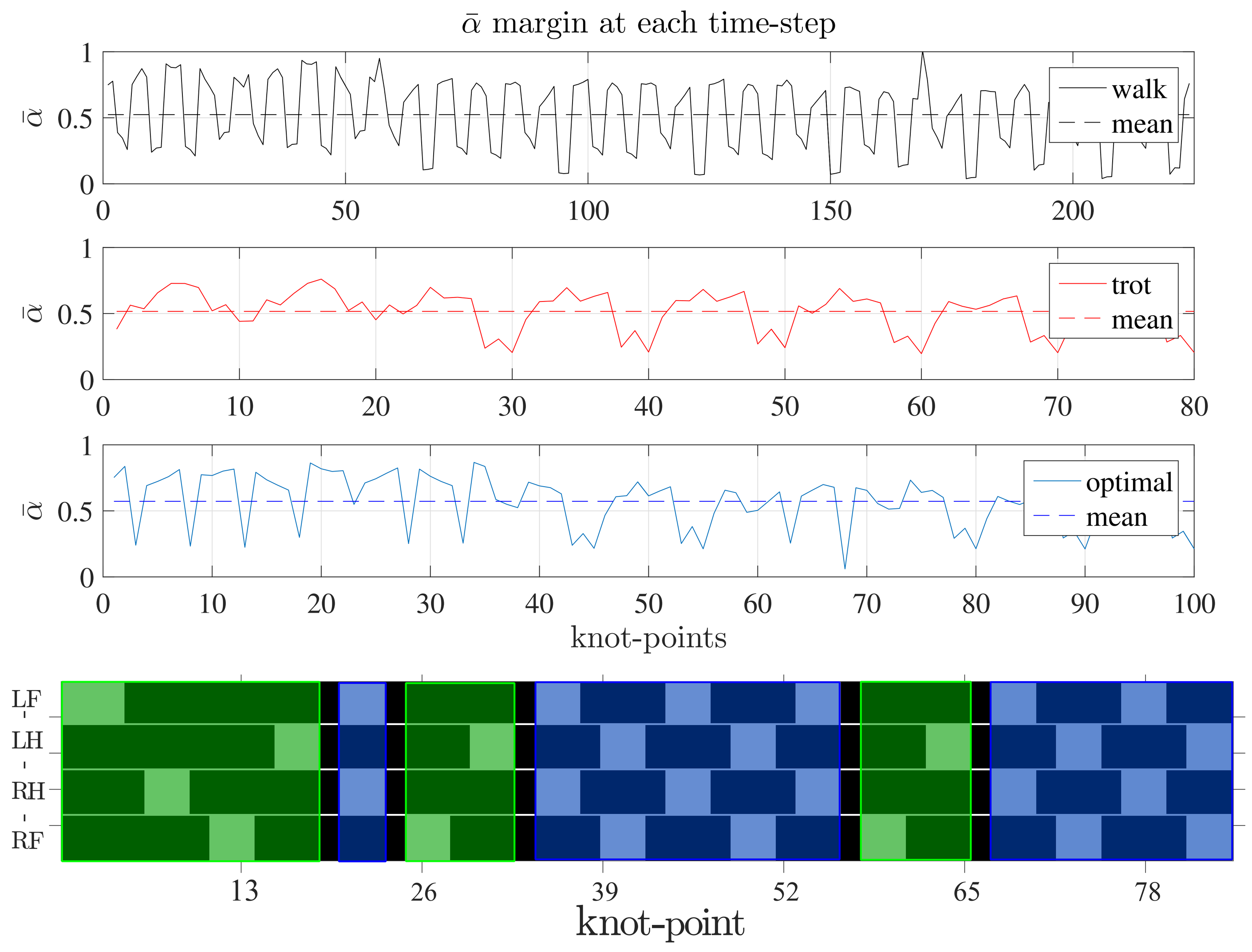
where $\mathbf{J}_{l,j} \in \mathbb{R}^{3\times 3}$ is the operational space foot Jacobian for the l^{th} leg at the j^{th} knot, and $\boldsymbol{\tau}_{max}$ are the joint torque limits of the leg.

Experimental Validation

Approximate Torque Constraints



Automatic Gait Discovery



Top: Normalized α margin for different gaits. *Bottom:* Resulting optimal gait sequence for navigating in roof-like terrain.

Computation Time

Experiment	convex surfaces	Gait	mean time (s)
Exp. 1	3	Walk	0.47
Exp. 2	3	Walk	0.64
Exp. 3	4	Walk	0.44
Exp. 4	3	Walk	0.48
Exp. 4	3	Trot	0.51
Exp. 4	3	Free	1.62

Computation time in an un-optimized Matlab code.

Conclusions

We have presented a novel approach for simultaneously planning contacts and motions on multi-legged robots based on MICP. Our approach is able to handle **complex terrain**, while also providing formal **robustness guarantees** on the plan and allows for **automatic gait discovery**. We employ both a state-of-the-art whole-body controller [1] and state estimation [6]. We demonstrate the approach's capabilities by traversing challenging terrains with the HyQ robot.

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